Wireless channel characterization

Part of the material is from:

W.C.Jakes Jr., *Microwave Mobile Communications*, John Wiley & Sons

T.S.Rappaport, *Wireless Communications – Principles & Practice*, Prentice Hall

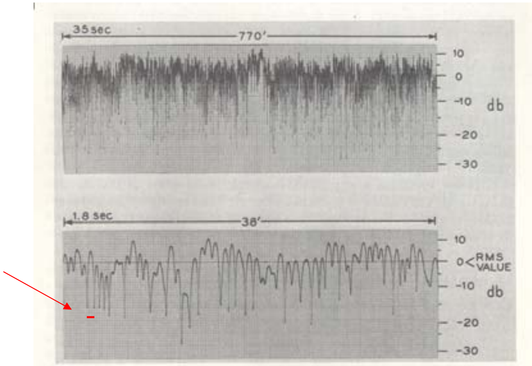
A.Goldsmith, *Wireless Communications*, Cambridge University Press

O.Andrisano, D.Dardari, *Appunti di Sistemi di Telecomunicazioni – Elementi di progetto di sistemi radiomobili*, Società Editrice Esculapio

Introduction

One of the most interesting applications of radio communications, that is communication between mobile people, has many impairments.

A microwave radio signal (450MHz-20GHz) with relative mobility tx-rx experiments variations in both amplitude and phase… fluctuations in amplitude of 40dB or more every half wavelength



λ/2≅18cm

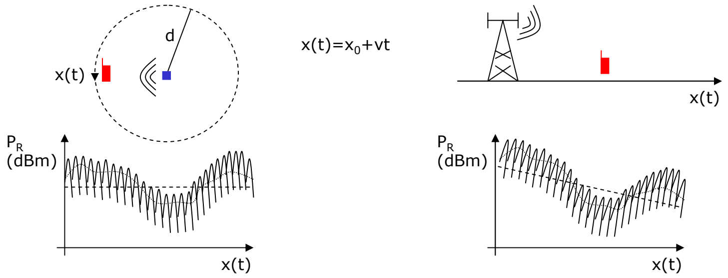
Figure 1.1-1 Typical received signal variations at 836 MHz measured at a mobile speed of 15 miles/hr. Records taken on the same street with different recording speeds.

Space / Time channel variations

Due to multiple users, mobility and environment dynamics, the mobile radio channel is impaired by noise, interference as well as time-varying fluctuations

System design requires statistical characterization of both disturbances and random channel space/time variations

Envelope variations are due to phenomena on different spatial/temporal scale



Envelope and phase variations

In the absence of obstacles and reflections the loss is given by a deterministic function of the distance (free-space path-loss).

In realistic channels both envelope and phase of rx signal fluctuate in space/time

The performance evaluation of
- non-coherent systems (that do not need phase recovery)
- ideal coherent systems (that are assumed perfectly recovering the phase)

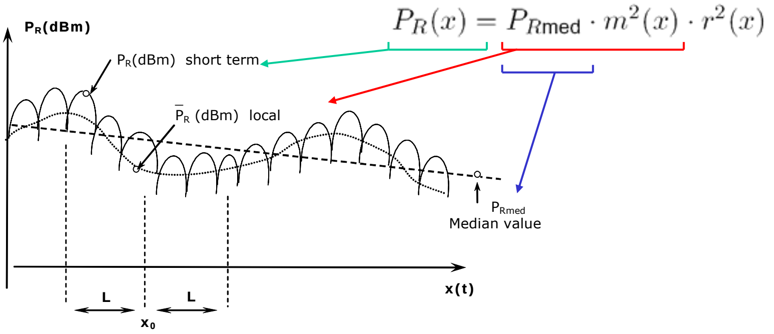
require the knowledge of envelope statistic

Small-scale and Large-scale effects

- multipath due to reflections close to the Rx → fast fading (small-scale propagation effects, i.e., variations in the order of the wavelength)

- obstacles between the Tx and the Rx → shadowing (large-scale effects, i.e., in the order of obstacles length 100-1000m outdoor, 10-100m indoor)

- deterministic attenuation → path-loss



$$P_R(x) = P_{R\text{med}} \cdot m^2(x) \cdot r^2(x)$$

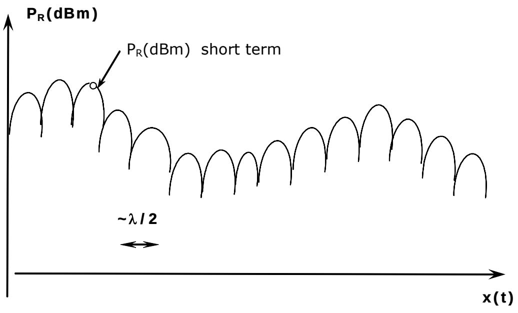
Short-term rx power

(mean over a carrier period)

- variations in the order of a wavelength

- mobility → frequency deviations due to Doppler-shifts → frequency spread → time-variations of the channel

- echoes → multipath → time spread → frequency variations of the channel

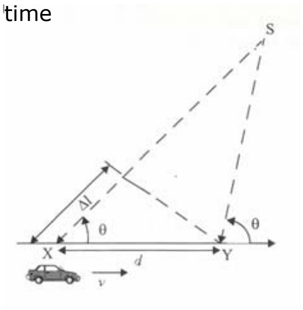


$\lambda \cong$ 33cm. @ 900MHz
$\lambda \cong$ 15cm. @ 2GHz

Doppler-shift

Mobility → time-varying distance → deviation in phase and frequency

In the presence of multipath all components are differently varied in time



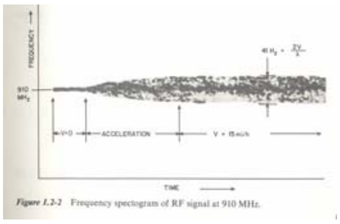


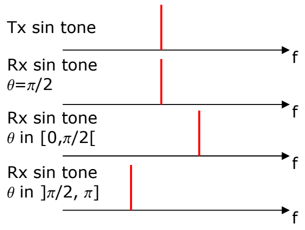
Figure 1.2-2 Frequency spectrogram of RF signal at 910 MHz.

$$\Delta\Phi = \frac{2\pi\ \Delta l}{\lambda} = \frac{2\pi\ v\ \Delta t \cos\theta}{\lambda}$$

$$f_d = \frac{\Delta\Phi}{2\pi\ \Delta t} = \frac{v}{\lambda}\ \cos\theta$$

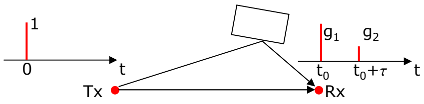
$$f_{d\text{max}} = v/\lambda$$ ← Maximum Doppler shift

$$B_d = 2v/\lambda$$ ← Doppler spread



Multipath example: 2-rays channel model

Two copies of the signal are received with different amplitudes, delays and phases.



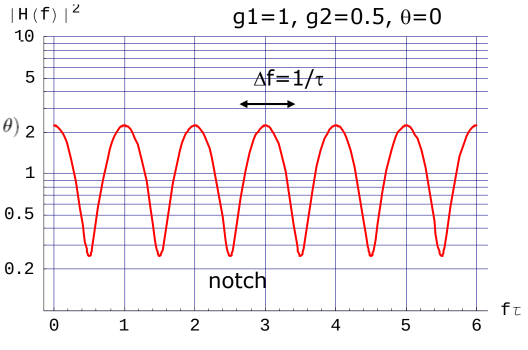
Ex.: t_0=0, g_1 and g_2 reals

$$h_0(t) = g_1\delta(t) + g_2\delta(t-\tau)\ e^{-j\theta}$$

$$H_0(f) = g_1 + g_2\ e^{-j(2\pi f\tau+\theta)}$$

$$|H_0(f)|^2 = g_1^2 + g_2^2 + 2g_1g_2\cos(2\pi f\tau+\theta)$$

Narrowband or wideband depending on **B**τ.



Paths profile for GSM

ETSI reccom. 05.05

Typical case for urban area (TUx): (12 tap setting)

Tap number	Relative time (μs)		Average relative power (dB)		doppler spectrum
	(1)	(2)	(1)	(2)	
1	0,0	0,0	-4,0	-4,0	CLASS
2	0,1	0,2	-3,0	-3,0	CLASS
3	0,3	0,4	0,0	0,0	CLASS
4	0,5	0,6	-2,6	-2,0	CLASS
5	0,8	0,8	-3,0	-3,0	CLASS
6	1,1	1,2	-5,0	-5,0	CLASS
7	1,3	1,4	-7,0	-7,0	CLASS
8	1,7	1,8	-5,0	-5,0	CLASS
9	2,3	2,4	-6,5	-6,0	CLASS
10	3,1	3,0	-8,6	-9,0	CLASS
11	3,2	3,2	-11,0	-11,0	CLASS
12	5,0	5,0	-10,0	-10,0	CLASS

The reduced TUx setting (6 taps) is:

Tap number	Relative time (μs)		Average relative power (dB)		doppler spectrum
	(1)	(2)	(1)	(2)	
1	0,0	0,0	-3,0	-3,0	CLASS
2	0,2	0,2	0,0	0,0	CLASS
3	0,5	0,6	-2,0	-2,0	CLASS
4	1,6	1,6	-6,0	-6,0	CLASS
5	2,3	2,4	-8,0	-8,0	CLASS
6	5,0	5,0	-10,0	-10,0	CLASS

Multipath delay spread and Coherence bandwidth

Echoes → time dispersion → variation in frequency

By evaluating multipath characteristics the following quantities are defined:

Max delay spread $\tau_{\max}$

RMS delay spread σ_τ

Coherence bandwidth (B_c): bandwidth over which two samples in frequency of channel response at a given time decorrelates more than a given level (e.g., 0.5)

$$B_c \simeq \frac{1}{\sigma_\tau} \qquad B_c \simeq \frac{1}{\tau_{\max}}$$

Doppler spread and Coherence time

Mobility → frequency deviation due to Doppler shift → frequency dispersion → variation in time of the channel

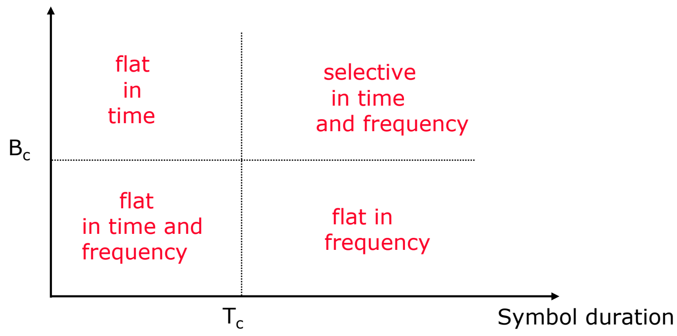
Max Doppler shift $f_{d\max} = \frac{v}{\lambda}$

Coherence time (T_c): time interval in which two samples of the channel at the same frequency, decorrelates below a given level (e.g., 0.5)

$$T_c \simeq \frac{9}{16\pi f_{d\max}} \qquad T_c \simeq \frac{1}{f_{d\max}}$$

Channel selectivity

Signal Bandwidth



flat in time

selective in time and frequency

B_c

flat in time and frequency

flat in frequency

T_c

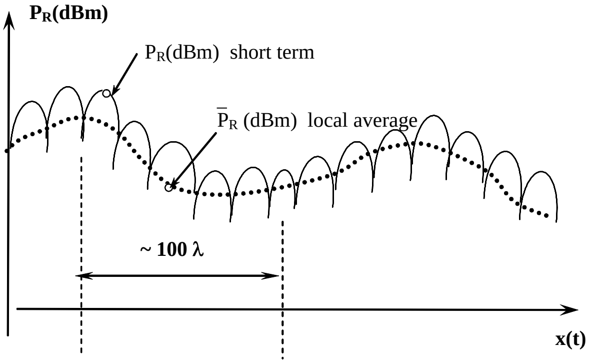
Symbol duration

Long-term rx power (local mean)

(mean evaluated in the space interval [x0-L,x0+L] with L=20λ - 40λ)

The effects of fast fading are averaged and fluctations are due to shadowing

$$\overline{P}_R(x_0) = < P_R(x) >_{[x_0-L,x_0+L]}$$



Short term fading statistic

$$P_R(x) = \overline{P}_R(x_0) \cdot r^2(x)$$

It will be demonstrated that under particular hps, that will be investigated, phases are uniformly randomly distributed U[-π,π], whereas the amplitude is distributed as:

Rayleigh
(NLOS)

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left\{-\frac{r^2}{2\sigma^2}\right\} \\ 0 \end{cases}$$

Rice
(LOS)

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left\{-\frac{r^2+A^2}{2\sigma^2}\right\} I_0\left(\frac{Ar}{\sigma^2}\right) & r \geq 0, \\ 0 & r < 0. \end{cases}$$

$$K = \frac{A^2}{2\sigma^2}$$

Inside fast fading…

$$s(t)=\mathbb{R}\left\{i(t)\exp[j2\pi f_0 t]\right\} \longrightarrow \boxed{h_0(t)} \longrightarrow s_r(t)=\mathbb{R}\left\{i_r(t)\exp[j2\pi f_0 t]\right\}$$

$$\boxed{i_r(t)=i(t)*h_0(t)=\sum_{n=0}^{N(t)-1}\alpha_n(t)i(t-\tau_n(t))\exp\{-j2\pi f_0\tau_n(t)-\Phi_{Dn}\}}$$

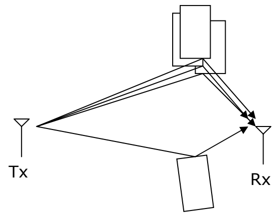
The number of paths N(t), amplitude, delay and phase of each of them and Doppler phase shift are randomly time varying

$$\Phi_{Dn}=\int 2\pi v/\lambda\cos\theta_n(\xi)d\xi$$

$$\boxed{h_0(t;\tau)=\sum_{n=0}^{N(t)-1}\alpha_n(t)\delta(\tau-\tau_n(t))\exp\{-j\underbrace{(2\pi f_0\tau_n(t)-\Phi_{Dn})}_{\Phi_n(t)}\}}$$

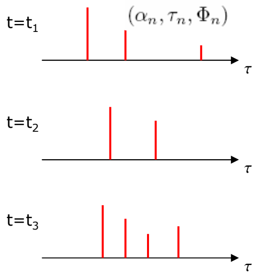
Resolvable paths

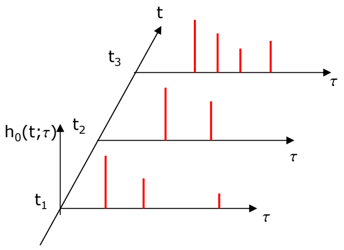
Single reflector and reflector cluster



Path *n* and *m* are resolvable if $|\tau_n-\tau_m|<<1/B$

N(t) is the number of resolvable path at time t. Nonresolvable paths are combined into a single component.





Phase to Envelope variations

For typical values of carrier frequency and delays (e.g., 1GHz and 50ns indoor, typically, greater delays in outdoor)

$$\Phi_n(t) = 2\pi f_0 \tau_n(t) - \Phi_{Dn}$$

→small delay variations imply great phase variations → constructiove and destructive addition of paths in small space → fading on the envelope

The impact of multipath depends on delay spread, τ_{max}, that is defined as the maximum among differences between delays and the delay of first path (sometimes referred to the mean delays)

$\tau_{max}(t)$ for time-varying channels

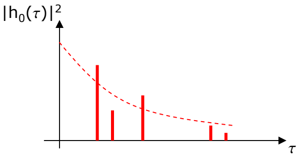
τ_{max} for indoor (10-1000ns), suburban (200-2000ns), urban (1-30us)

$\tau_{max} << 1/B$	nonresolvable paths	→ narrow-band fading model
$\tau_{max} > 1/B$	resolvable paths	→ wide-band fading model

Power Delay Profile

Paths do not have the same relevance; in particular, they are negligible in delay spread evaluation when their related power is below the noise floor.

By locally averaging (in space/time) the squared envelope of the equivalent low pass impulse response of the channel, a time invariant figure is obtained, that is the channel power delay profile



$$\overline{\tau} = \frac{\sum_k a_k^2 \, \tau_k}{\sum_k a_k^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \, \tau_k^2}{\sum_k a_k^2}$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \overline{\tau}^2}$$

rms delay spread

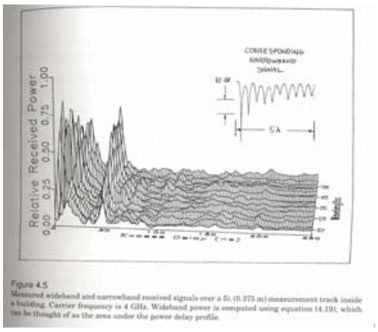


Figure 4.5
Measured wideband and narrowband received signals over a 5λ (0.375 m) measurement track inside a building. Carrier frequency is 4 GHz. Wideband power is computed using equation (4.19), which can be thought of as the area under the power delay profile.

Narrowband fading model - 1

$\tau_{max} << 1/B$ nonresolvable paths

$$s_r(t) = \mathbb{R}\left\{\sum_{n=0}^{N(t)-1} \alpha_n(t) i(t-\tau_n(t)) \exp\{-j\Phi_n(t)\} \exp\{j2\pi f_0 t\}\right\}$$

$$s_r(t) \approx \mathbb{R}\left\{i(t) \exp\{j2\pi f_0 t\} \sum_{n=0}^{N(t)-1} \alpha_n(t) \exp\{-j\Phi_n(t)\}\right\}$$

On the entire signal bandwidth the same behavior of the central frequency → characterization supposing a sin tone at f_0 (if ampl. 1 anf initial phase 0 then i(t)=1).

$$s_r(t) \approx \underbrace{\mathbb{R}\left\{\exp\{j2\pi f_0 t\} \sum_{n=0}^{N(t)-1} \alpha_n(t) \exp\{-j\Phi_n(t)\}\right\}}_{r_I(t)\cos(2\pi f_0 t) - r_Q(t)\sin(2\pi f_0 t)}$$

Narrowband fading model - 2

$$s_r(t) \approx \underbrace{\mathbb{R}\left\{\exp\{j2\pi f_0 t\} \sum_{n=0}^{N(t)-1} \alpha_n(t) \exp\{-j\Phi_n(t)\}\right\}}_{r_I(t)\cos(2\pi f_0 t) - r_Q(t)\sin(2\pi f_0 t)}$$

Hps.:

dense multipath → N(t)>>1 → for the CLT the $r_I(t)$ and $r_Q(t)$ are jointly Gaussian

N-LOS

$\alpha_n(t)$, $\tau_n(t)$, $f_{Dn}(t)$ slowly varying → const in the observation interval to α_n, τ_n, f_{Dn}

$\Phi_n(t)$ fast → $U[-\pi,\pi]$

…

Narrowband fading model - 3

$s_r(t)$ is Complex Gaussian with 0 mean and $r_I(t)$ and $r_Q(t)$ are uncorrelated

$r_I(t)$ and $r_Q(t)$, thus $s_r(t)$, wide-sense stationary (WSS)

$$A_{r_I}(t;t+\tau) = \mathbb{E}\{r_I(t)r_I(t+\tau)\} = 1/2\ \sum_n \mathbb{E}\{\alpha_n^2\}\cos(2\pi v\tau\cos(\theta_n/\lambda)) = A_{r_I}(\tau)$$

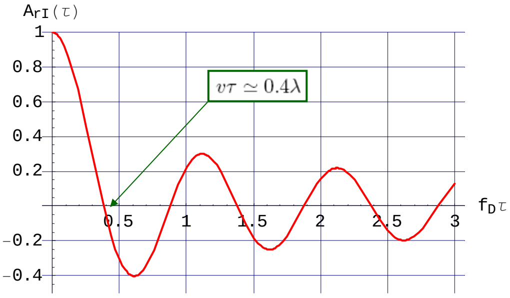
$$A_{r_Q}(t;t+\tau) = A_{r_I}(\tau)$$

Hps.

Uniform scattering i.e., N infinity and

$$\overline{P}_r = (N/2)\mathbb{E}\{\alpha^2\} \quad \Delta\theta = 2\pi/N \quad \theta_n = n\Delta\theta$$

$$\boxed{\begin{aligned} A_{r_I}(\tau) &= \overline{P}_r\ J_0(2\pi f_{d\text{max}}\tau) \\ A_{r_I r_Q}(\tau) &= 0 \end{aligned}}$$



$$J_0(x) = 1/\pi\ \int_0^\pi \exp[-jx\cos\theta]d\theta$$

Narrowband fading model - 4

The fast fading decorrelates after 0.4λ (spacing for multiple antennas)

By Fourier-transforming the autocorrelation function and noting that r_I and r_Q are uncorrelated…

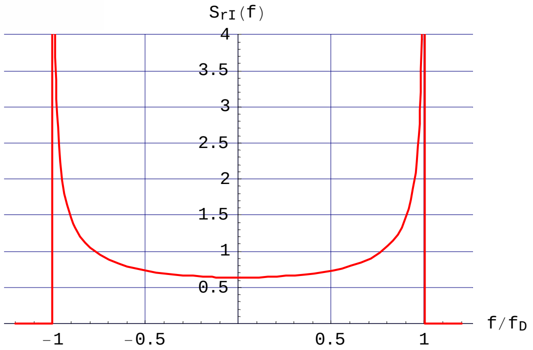
$$S_{rI}(f) = S_{rQ}(f) = \begin{cases} \frac{2\overline{P}_r}{\pi f_{d\text{max}}}\ \frac{1}{\sqrt{1-(f/f_{d\text{max}})^2}} & |f| \leq f_{d\text{max}}, \\ 0 & \text{otherwise.} \end{cases}$$

$$S_r(f) = 1/4S_{rI}(f-f_0) + 1/4S_{rI}(f+f_0)$$

Asynt not present in reality
(no uniform scattering)

It can be seen as the pdf of
random freq deviation due to
Doppler effects → *simulations*

Area= $\overline{P}_r$



Narrowband fading model - 5

At time t, both r_I and r_Q are ~N(0,σ^2) with $\overline{P}_r = (N/2)\mathbb{E}\{\alpha^2\} = 2\sigma^2$

It follows that r_I+jr_Q is CN distributed with envelope Rayleigh distributed and phase ~U[$-\pi$, π]

$$\boxed{f_R(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left\{-\frac{r^2}{2\sigma^2}\right\} & r \geq 0, \\ 0 & r < 0. \end{cases}}$$

Rayleigh fading (N-LOS)

If there is a dominant path (usually due to LOS), then one of the two components does not have zero mean and the envelope results distributed as

$$\boxed{f_R(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left\{-\frac{r^2+A^2}{2\sigma^2}\right\} I_0\left(\frac{Ar}{\sigma^2}\right) & r \geq 0, \\ 0 & r < 0. \end{cases}}$$

Rice(ean) fading (LOS)

$$\boxed{K = \frac{A^2}{2\sigma^2}}$$

Rice factor

Narrowband fading model - 6

Comparison with
measurement campaigns…

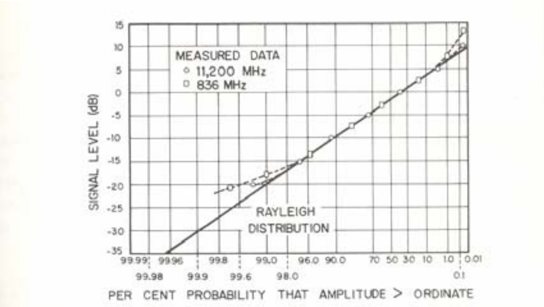


Figure 1.1-5 Cumulative probability distributions for 836 and 11,200 MHz.

Figure 2.2-4 Approximate region covered in smaller area model.

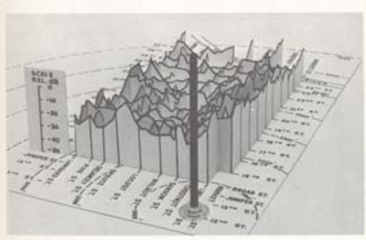


Figure 2.2-5 Model of fine detail test area.

SNR, γ, pdf for some common fading channels

Fading type	$f_\gamma(\gamma)$ (0 for γ<0)	
Rayleigh	$\frac{1}{\bar{\gamma}}\ \exp(-\gamma/\bar{\gamma})$	
Rice	$\frac{1+K}{\bar{\gamma}}\ \exp(-K-(1+K)\gamma/\bar{\gamma})$	$0 \le K$
Nakagami-m (from meas.)	$\frac{m^m\ \gamma^{m-1}}{\bar{\gamma}^m\Gamma(m)}\ \exp(-m\gamma/\bar{\gamma})$	$\frac{1}{2} \le m$

Mean bit error probability (averaged over fast fading)

In many applications (e.g., voice) the perceived QoS at the final user is related to the error rate experimented in a time interval of few seconds → fast fading fluctuations are averaged in that period → system design with QoS requirements on the mean BEP, that is

$$\overline{P}_b(\bar{\gamma}) = \mathbb{E}\left\{P_\text{b}\right\} = \int_0^{+\infty} f_\gamma(\xi) P_b(\xi)\, d\xi$$

Ex.: BPSK coerente

$$P_\text{b}(\gamma) = \frac{1}{2}\text{erfc}\sqrt{\gamma}$$

$$\overline{P}_b(\bar{\gamma}) = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

Wideband fading model

C…

Median rx power

$$\mathbb{P}\left\{\overline{P}_R(x) < P_{R\text{med}}\right\}_{L\simeq 1000\lambda} = 50\%$$

On a space/time scale for which effects of both fast fading and shadowing are averaged, the mean received power is deterministic. Its value is a function of the link distance → path-loss → A_I

C_r ← link-budget

$$\overline{P}_R(x) = P_{R\text{med}} \cdot m^2(x)$$

shadowing

Shadowing

It is statistically described by a log-normal pdf, that is $m^2(x)$ in dB is Gaussian with mean 0 and variance σ_{dB}^2

Note: σ_{dB}^2 is not the variance in dB scale but the linear value of the variance of $m^2(dB)$

i.e., the pdf of m^2 is given by:

$$f_{\overline{\gamma}}(w) = \begin{cases} \frac{\nu}{\sqrt{2\pi}\sigma w} \exp\left[-\frac{(10\log_{10} w - \mu_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}\right] & w \geq 0, \\ 0 & \text{otherwise,} \end{cases} \qquad \nu = 10/\ln 10$$

QoS-based outage

QoS-based outage probability (bit error outage for digital comm systems)

Ex.1:
QoS=inst. BEP

$$P_{\mathsf{o}} = \mathbb{P}\left\{P_{\mathsf{b}} \geq P_{\mathsf{bx}}\right\} = \int_0^{\gamma^\star} f_\gamma(\xi)\, d\xi$$

Rayleigh fading only

$$P_{\mathsf{o}}(\overline{\gamma}) = 1 - \exp\{\gamma^\star/\overline{\gamma}\}$$

Ex.2:
QoS=mean BEP

$$P_{\mathsf{o}} = \mathbb{P}\left\{\overline{P_{\mathsf{b}}} \geq \overline{P_{\mathsf{bx}}}\right\} = \int_0^{\overline{\gamma}_{\mathsf{x}}} f_{\overline{\gamma}}(\xi)\, d\xi$$

Log-N shadowing

$$P_{\mathsf{o}} = Q\left(\frac{\mu_{\mathsf{dB}} - 10\log_{10}\overline{\gamma}_{\mathsf{x}}}{\sigma_{\mathsf{dB}}}\right)$$